The structure of a source from interferometry measurements in heavy ion collisions

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Abstract

Some experimental results of correlation functions in Bose-Einstein (B-E) interferometry measurements exhibit a non smooth behaviour - oscillations. Possible origin of such a behaviour in non-trivial spatial distribution of the source is considered here. A method for obtaining additional information about the structure of source emitting identical particles is suggested. It is shown that this information could be extracted by a one dimensional fourier analysis of the filtered correlation function. Further possible enhancement of the method is sketched. Some technical aspects of the proposed technique are discussed.

Part1: Introduction

The idea of measuring properties of a source of identical particles by correlation experiments originated in radioastronomy. The angular diameter of the star Sirius was successfully determined by HBT method [1] in agreement with predictions. In experiments on antiproton-proton annihilation in 1959 the correlation effect in like-sign two pion angular distribution was discovered. This effect was interpreted in [2] as a result of B-E interference of the wave functions of emitted pions. In the simplest, sufficient for our purposes form the correlation function for a static source can be up to the normalization factor expressed as:

$$C(\Delta\vec{p}) \simeq \int\int f(\vec{x}_1) f(\vec{x}_2) |\Psi_{12}(\vec{x}_1, \vec{x}_2, \vec{p}_1, \vec{p}_2)|^2 d^3\vec{x}_1 d^3\vec{x}_2 \tag{1}$$

where $\vec{p}_1, \vec{p}_2$ are momenta of emitted identical bosons and $f(\vec{x})$ describes the geometrical distribution of the source. The squared absolute value of Ψ_{12} is:

$$|\Psi_{12}|^2 = \frac{1}{2}|e^{i\vec{p}_1\vec{x}_1}e^{i\vec{p}_2\vec{x}_2} + e^{i\vec{p}_1\vec{x}_2}e^{i\vec{p}_2\vec{x}_1}|^2 = 1 + \cos(\Delta\vec{x}\cdot\Delta\vec{p}) \tag{2}$$

and it depends only on the relative momentum of the emitted bosons $\Delta\vec{p} = \vec{p}_1 - \vec{p}_2$ and on the relative position of the emission points $\Delta\vec{x} = \vec{x}_1 - \vec{x}_2$. For a Gaussian source $f(\vec{x}) \simeq e^{-(x^2/R^2)}$ the correlation function (1) can be calculated analytically:

$$C(\Delta\vec{p}) \simeq \int\int |\Psi_{12}|^2 e^{-(\vec{x}_1^2 - \vec{x}_2^2)/R^2} d^3\vec{x}_1 d^3\vec{x}_2 = 1 + e^{-|\vec{p}_1 - \vec{p}_2|^2 R^2/2} \tag{3}$$

By fitting experimental data for the correlation function with (3) an approximate radius of the spherical source can be determined. A more complicated Gaussian parametrization

of the correlation function [3] separates the longitudinal, outward and sideward dimensions of the source. Statistical errors of correlation function data in heavy ion collisions (HIC) experiments became smaller but the interesting structure of the correlation function in the higher relative momenta region is still present in the recent results [4]. The influence of Coulomb final state interaction leads to the more interesting shape of the theoretical correlation function for the like sign charged particles [5,6], but it is not rich enough to explain any oscillations of the experimental correlation function in the $\Delta p > 100 MeV/c$ region[1]. Perhaps, it could be possible to reveal some information about the source from this behaviour of the correlation function.

In the following section we define the longitudinal correlation function which is subsequently used in the definition of distance spectrum of the source in Sec.3. In Sec.4 we discuss the reconstruction of the distance spectrum function from the longitudinal correlation function and some technical aspects of this procedure. In Sec.5. we present a simple example as the illustration of the method. In the last part we consider the possibility to apply a tomography techniques based on the approach presented here.

In the whole paper we deal only with the influence of the spatial distribution of source on the correlation function. The effects of the final state interactions and the coherence of emission are not considered here. It is assumed that they do not affect the principles of the method presented.

Part2: Longitudinal Correlation Function

Let us consider a spatial static three-dimensional source emitting pairs of boson particles. Because of B-E interference effect, the emission amplitude of a pair of particles with relative momentum $\Delta\vec{p} = \vec{p}_2 - \vec{p}_2$ depends on the relative position of the emission points. We can decompose the relative momentum of the detected pair into transverse and longitudinal components $\Delta\vec{p} = \Delta\vec{p}_t + \Delta\vec{p}_l$, where the longitudinal direction is chosen along the beam in the HIC experiment. Thus we have:

$$\Delta\vec{p} \cdot \Delta\vec{x} = (\Delta\vec{p}_l + \Delta\vec{p}_t) \cdot (\Delta\vec{x}_l + \Delta\vec{x}_t) = \Delta\vec{p}_l \cdot \Delta\vec{x}_l + \Delta\vec{p}_t \cdot \Delta\vec{x}_t \tag{4}$$

Let us choose events with a small transverse component of relative momentum

$$|\Delta\vec{p}_t| << |\Delta\vec{p}_l| \tag{5}$$

Then the transverse separation of emission points does not have influence on the correlation function for our set of data. An analogous consideration can be made for the set of events filtered by the condition

$$|\Delta\vec{p}_t| >> |\Delta\vec{p}_l| \tag{6}$$

or one can choose the sideward and outward selections used e.g. in [3] but we shall now concentrate on the correlation function for data events fulfilling the condition (5) - the longitudinal correlation function. In this case after the substitution (4) into (1) and assuming (5) we have:

$$\begin{aligned} C(\Delta\vec{p}) &= \int\int f(\vec{x}_1) f(\vec{x}_2)(1 + \cos(\Delta\vec{x} \cdot \Delta\vec{p})) d^3\vec{x}_1 d^3\vec{x}_2 = \\ &= \int\int F(x_l^1) F(x_l^2)(1 + \cos(\Delta x_l \cdot \Delta p_l)) dx_l^1 dx_l^2 \end{aligned} \tag{7}$$

[1] The oscillations in theoretical shapes of the correlation functions for photons can be found in the paper [7] (but see also [9])

where $F(x_l) = \int f(x_l, \vec{x}_t) d^2\vec{x}_t$.

This means, that our $3D$ source produces $C(\Delta p_l)$ like a one dimensional source with the distribution $F(x_l)$.

Part3: Distance Spectrum of the Source

Let us find how the longitudinal correlation function $C(\Delta p_l)$ looks like for a two-point source (see Fig.1a). This two point source has a distribution function $F(x) = 1/2(\delta(x-a) + \delta(x-b))$. The longitudinal correlation function then is:

$$C2(d, \Delta p_l) = \int F(x_1)F(x_2)(1 + \cos(\Delta p_l(x_1 - x_2)))dx_1 dx_2 = 1 + \cos(\Delta p_l \cdot d) \tag{8}$$

where $d = |a - b|$ is the separation of emission points. For a source with the distribution function $F(x) = P_1 \cdot \delta(x-x_1) + P_2 \cdot \delta(x-x_2) + P_3 \cdot \delta(x-x_3)$ (Fig.1b), where $P_1 + P_2 + P_3 = 1$ the result is:

$$C(\Delta p_l) = \Theta + P_{12} \cdot C2(d_{12}, \Delta p_l) + P_{13} \cdot C2(d_{13}, \Delta p_l) + P_{23} \cdot C2(d_{23}, \Delta p_l) \tag{9}$$

where $P_{ij} = P_i \cdot P_j$, $d_{ij} = |x_i - x_j|$ and Θ is a constant shift which will not be important in our further calculations. Based on this it seems appropriate to define the Distance Spectrum of a source. Each source emitting pairs of particles has its 'squeeze' - Distance Spectrum. It is the probability distribution for the emission of the pair in the distance D of emission points. For the two-point source the distance spectrum is $S(D) \simeq \delta(D - d_0)$ (see Fig.1a), for our three-point source (8) the distance spectrum is $S(D) \simeq P_{12}\delta(D-d_{12}) + P_{13}\delta(D-d_{13}) + P_{23}\delta(D-d_{23})$ (see Fig.1b). In general the distance spectrum for a one dimensional static source can be expressed as:

$$S(D) = \int f(x_1)f(x_2)\delta(D - |x_1 - x_2|)dx_1 dx_2 \tag{10}$$

Now we can write the longitudinal correlation function produced by a source with distance spectrum $S(D)$:

$$C(\Delta p_l) = \Theta' + \int S(D)C2(D, \Delta p_l)dD \tag{11}$$

A distance spectrum does not contain complete information about the spatial distribution of a source. Slightly different spatially distributed sources can have the same $S(D)$ (see also Sec.5.). The distance spectrum of another point-like source is shown in Fig.1c .

Part4: Inverse Transformation

Expression (1) for the correlation function can be after the formal integration written in the form:

$$C(\Delta\vec{p}) = 1 + |\tilde{f}(\Delta\vec{p})|^2 \tag{12}$$

where $\tilde{f}(\Delta\vec{p}) = \int e^{i\Delta\vec{p}\cdot\vec{x}} f(\vec{x}) d^3\vec{x}$. The absolute value in expression (12) destroys the phase information in the fourier picture $\tilde{f}(\Delta\vec{p})$ of source distribution. This breaks the possibility to perform the inverse fourier transformation in order to get $f(\vec{x})$.

In this section we shall show that the remaining information in $|\tilde{f}(\Delta\vec{p})|$ has certain physical meaning and that its extraction is at least theoretically possible.

The form of expression (11) is sufficiently simple. Therefore one could consider a possibility to gain the function $S(D)$ from $C(\Delta p_l)$. As we shall see, the $S(D)$ is just the information contained in $C(\Delta p_l)$. According to (11) and (8) we have

$$C(\Delta p_l) = \Theta'' + \int S(D)\cos(\Delta p_l \cdot D) dD \tag{13}$$

The constant shift factor can be separated from the correlation function data and so we can write:

$$\tilde{C}(\Delta p_l) = \int_0^\infty S(D)\cos(\Delta p_l \cdot D) dD \tag{14}$$

where $\tilde{C}(\Delta p_l)$ is the correlation function with the shift factor removed. Expression (14) is fourier transformation and we can try to perform the inverse fourier transformation in order to obtain $S(D)$ from experimental data.

$$S(D) \simeq \int_0^\infty \tilde{C}(\Delta p_l)\cos(\Delta p_l \cdot D) d(\Delta p_l) \tag{15}$$

Because of the fact, that we measure $C(\Delta p_l)$ only in discrete points it is necessary to replace integration by summation.

$$S_F(D) \simeq \sum_i^{\text{points}} \tilde{C}(\Delta p_l^i)\cos(\Delta p_l^i \cdot D)(\Delta p_l^{i-1} - \Delta p_l^i) \tag{16}$$

However a more relevant problem is that the integration region in (15) includes the values of Δp_l which cannot be measured (high Δp_l). The cut-off in Δp_l is present due to energy conservation considerations and due to the growth of statistical errors in a high Δp_l region. The influence of such a cut-off on the result of Fourier transformation leads to the oscillations in the resulting $S(D)$ (see Fig.2a) regardless of the number of measured points in the region $(0, \Delta p_{\text{max}})$. Similar problems occur e.g. in digital FIR[2] filter design. Fortunately these oscillations can be suppressed by the Method of Windows [8]. The method is based on the simple multiplication of the original function to be fourier transformed by the Window Function, which suppresses the values of the original function near the cut-off (see Fig.2b).

$$S_W(D) \simeq \sum_i^{\text{points}} W(\Delta p_l^i)\tilde{C}(\Delta p_l^i)\cos(\Delta p_l^i \cdot D)(\Delta p_l^{i-1} - \Delta p_l^i) \tag{17}$$

There are several types of the Window Function. In Fig. 2b a simple Gaussian function is used. The amplitude of the oscillations depends also on the shape of the exact (without cut-off) result of the Fourier transformation. The exact result of Fourier transformation of the function in Fig. 2a without cut-off would be Dirac delta function at the point $d_0 = 3$fm. Such a shape leads to a big oscillations.

The influence of statistical errors of correlation function data to the resulting distance spectrum which is crucial for the eligibility of the method is not considered in this paper.

Part5: Structured Time Dependent Source

As we have already mentioned the sources with the same $S(D)$ can be different, but in spite of that, having the distance spectrum of a source we can make some conclusions about

[2]FIR = finite duration impulse response

the source. At least the average radius

$$\bar{R} = \int D \cdot S(D) dD \tag{18}$$

and maximal radius $R_{\max} = D_{\max}$ in $S(D)$ can be found from $S(D)$.

Let us consider a three-dimensional source consisting of two separated regions (Fig.3c). Both regions A and B contribute to distance spectrum of a source in longitudinal (5) analysis in the interval $0 < D < R$. When one of the particles comes from A and the second from B they can be emitted in a distance interval $K - R < D < K + R$. The distance spectrum $S(D)$ in the interval $R < D < K - R$ is suppressed as much as the parts A and B are located. Original spectra $S_{MC}(D)$ for such a source computed by Monte-Carlo simulation, the correlation functions calculated according expression (11) at 200 points in the $\Delta p_l < 2$GeV interval and the results of inverse transformation (16) for different separations K of parts A, B of the source are in figures 3a, 3b, 3c.

The real experimental situation is however a time dependent distribution of the source. The general expression for the correlation function in this case is much more complicated than expression (1):

$$C(\Delta p) = \int \int w(x_1, \frac{p_1 + p_2}{2}) w(x_2, \frac{p_2 + p_1}{2})(1 + \cos(\Delta x^\mu \Delta p_\mu)) d^4x_1 d^4x_2 \tag{19}$$

However the influence of the spatial structure of the source could still demonstrate itself in the oscillations of the correlation function at the higher relative momentum region.

Following the simple approach based on (1) in the case of the time evolution we have to take into consideration that the bosons emitted in different times do interfere. This means, that the same region of the hadron gas or quark gluon plasma emitting the bosons in two different times t_1 and t_2 at the positions $\vec{x}_1$ and $\vec{x}_2$ contributes to our distance spectrum of the source like a two different regions emitting the bosons at the same time but in separate points $\vec{x}_1$ and $\vec{x}_2$. Therefore an important question arises: How big can be the time difference of the processes of emission for interfering bosons ?

Let us denote the duration of the process of emission of a single boson as τ_0. As an upper limit for τ_0 the formation time of the emitted boson which is of the order of 1fm/c [11] could be chosen. It seems to be reasonable to require a time overlapping of the processes of emission for the interfering bosons in the B-E interference phenomenon. Assuming the Lorentz dilatation factor γ for τ_0 of emitted boson we obtain the rough restriction on the time difference Δt of emissions of the interfering bosons in the form:

$$\Delta t < \tau_0 \cdot \gamma \tag{20}$$

For a typical momentum of pions produced in heavy ion collisions the time Δt is long compared to m_π^{-1} what is necessary for the influence of B-E interference on correlation function [10].

Different spatial distributions in a different time intervals lead to different distance spectra $s(D,t)$. In our simple approach the correlation function for a time dependent source could be expressed for $\tau_0 \to 0$ as:

$$\tilde{C}(\Delta p_l) = \int \int_0^\infty s(D,t) \cos(\Delta p_l \cdot D) dD dt = \int_0^\infty \left[\int s(D,t) dt \right] \cos(\Delta p_l \cdot D) dD \tag{21}$$

but for $\tau_0 \neq 0$ the information about the $s(D,t)$ in the correlation function is smudged with the uncertainty proportional to Δt given by (20).

It is clear, that by the inverse transformation (15) we can obtain only $S(D)$, the result of the time integration in (21). A combination of the results obtained for different kinds of identical particles emitted in different stages of the evolution of the heavy ion collision or the results obtained for filtered events which can be produced only during characteristic and short time intervals of the evolution process could bring some information about $s(D,t)$.

Part6: B-E Tomography

In this section we shall deal with the set of events fulfilling condition (6) - transversal events. Analogously to Sec.2. the longitudinal separation of emission points does not have influence on the transversal correlation function[3] now, but condition (6) is not so selective as condition (5). We have a freedom in azimuthal direction of $\Delta\vec{p}_t$ and the emission points lie in the plane orthogonal to the beam direction (see Fig.4). It is possible to apply some further condition to have the same one-dimensional situation like in the preceding sections. For example the condition

$$\Delta\vec{p}_t \cdot \vec{n}_\varphi >> |\Delta\vec{p}_t \times \vec{n}_\varphi| \tag{22}$$

where $\vec{n}_\varphi$ is a normalized vector orthogonal to the beam direction (see Fig.4), selects the pairs with relative momentum in the direction of vector $\vec{n}_\varphi$.

However the cylindrical symmetry of the radiating volume in HIC, at least for the central collisions, instigates us to try to gain the whole two-dimensional information from $C(\Delta\vec{p}_t)$. Let us define two-dimensional distance spectrum as:

$$S(\vec{D}) = \int\int f(\vec{x}_1) f(\vec{x}_2) \delta(\vec{D} - (\vec{x}_1 - \vec{x}_2)) d^2\vec{x}_1 d^2\vec{x}_2 \tag{23}$$

$S(\vec{D})$ is the probability distribution of emission of the pair of particles from the points at relative position $\vec{x}_1 - \vec{x}_2 = \vec{D}$ on the transversal plane. Then for our transversal correlation function

$$\tilde{C}(\Delta\vec{p}_t) = \int f(\vec{x}_1) f(\vec{x}_2) \cos(\Delta\vec{p}_t \cdot \Delta\vec{x}) d^2\vec{x}_1 d^2\vec{x}_2 \tag{24}$$

we can write

$$\tilde{C}(\Delta\vec{p}_t) = \int S(\vec{D}) \cdot \cos(\Delta\vec{p}_t \cdot \vec{D}) d^2\vec{D} \tag{25}$$

what can be proven by inserting (23) into (25).

For a set of transversal events fulfilling the condition (22) specified by the angle φ the correlation function is:

$$\begin{aligned} \tilde{C}(\Delta p_\varphi) &= \int\int S(\vec{D}) \cos(\Delta\vec{p}_\varphi \cdot (\vec{D}_\varphi + \vec{D}_\varphi^\perp) dD_\varphi dD_\varphi^\perp = \\ &= \int \left[\int S(\vec{D}) dD_\varphi^\perp\right] \cos(\Delta p_\varphi \cdot D_\varphi) dD_\varphi = \\ &= \int S_\varphi(D_\varphi) \cdot \cos(\Delta p_\varphi \cdot D_\varphi) dD_\varphi \end{aligned} \tag{26}$$

where the index " $_\varphi^\perp$" describes the orthogonal and the index " $_\varphi$" parallel direction to the vector $\vec{n}$. Thus by the inverse transformation (15) we can obtain the projection $S_\varphi(D)$ of the two-dimensional distance spectrum $S(\vec{D})$ to the direction determined by the angle φ. This can be done for any direction, for example in 2° steps of our angle φ.

[3] the correlation function for the transversal set of events

From such a set of projections one can get the whole $S(\vec{D})$ by Radon Transformation which is used in tomography [13].

It is hard to imagine that this technique could be applied to the interferometry experimental data in heavy ion collisions. However in the future HIC experiments e.g. at LHC the multiciplities/event could allow to do something close to the ideas presented here. For the final decision whether this tomography method is eligible or not a careful and precise estimate of statistical errors is necessary. Both the subjects - the description of Radon Transformation and the analysis of statistical errors are not included in this paper.

One can imagine that the proposed two-dimensional tomographic method is applicable and even more, that a time dependent three-dimensional distance spectrum can be somehow gained. The question arises: What could we see from $S(\vec{D})$ or $S(\vec{D},t)$? Author thinks that it could be possible to see some signatures of the phase transition in this kind of analysis. In cosmology the phase transition as the only explanation of the inhomogeneities in the distribution of matter in universe is used. We can hope that the phase transition in HIC could lead to the formation of the inhomogeneities - the bubbles in the volume of the collision. These bubbles emitting particles in a different strength than the surrounding medium could be seen in our distance spectra.

Part7: Summary

We have studied the influence of the spatial distribution of a source on the correlation function in a set of longitudinal (5) events. It was found, that spatially complicated structure of the source can lead to oscillations of the correlation function. It is shown that from the longitudinal correlation function the distance spectrum of a source in the longitudinal direction can be derived. The radius of the source, which is used to characterize the longitudinal size of the collision volume is expressed as a simple integral of the distance spectrum. Some technical aspects of the suggested method are discussed. The problem of the time dependence of spatial distribution of a source is also considered. An analogous method for transversal set of events and the possibility to use the tomography method in this approach are considered. The proposed technique could evoke some ideas how to enhance the interferometry methods in the future heavy ion collisions experiments.

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Figure captions

[Fig. 1a,1b,1c] The point-like sources and their distance spectra obtained by Monte-Carlo simulation in the agreement with the analytical probability calculation.

[Fig. 2a,2b] The correlation function $C2(\Delta p)$ for two-point source (distance $d_0 = 3$fm) and the result of the inverse transformation without (2a) and with (2b) the Method of windows.

[Fig. 3a,3b,3c] The distance spectrum $S_{MC}(D)$ calculated by Monte-Carlo simulation, the corresponding correlation function $\tilde{C}2(\Delta p)$ at 200 points and a result of inverse transformation (15) for the three different separations of the sources A, B.

[Fig. 4] The plane transversal to the beam direction and the vector $\vec{n}$ with direction determined by the angle φ. For the central collision the zero angle direction can be chosen randomly. For a non central collision an asymmetry in azimuthal direction in the distributions of the produced particles [12] could be used to determine the zero angle direction.